

## A review of basic principles of electricity & physics

### *Introduction*

While this book is not intended as a comprehensive course in electricity and magnetism there are a few principles that are so ubiquitous in functional magnetic resonance that they appear repeatedly throughout this text and therefore warrant this brief overview.

### *Charge*

Electrical charge is considered to be a fundamental property of materials. Physicists recognize that charge exists in only two forms, positive and negative, and that it is quantal in nature, with the smallest amount of charge being that of a single electron or proton, each being exactly 1 unit of negative or positive charge, respectively. A single unit of charge is extremely small, of course, and charge is more commonly measured in units of **Coulombs**, equivalent to about  $6.242 \times 10^{18}$  unit charges. Positive and negative charges exhibit a strong attractive force, whose magnitude is proportional *inversely* to the square root of the distance that separates them. In its most stable state, bulk matter has a *net* charge of zero, meaning that it contains an identical number of positive and negative charges.

### *Voltage*

When charges become separated by distance, the presence of an attractive force between implies an increase in potential energy, which is released when the charges are moved together. This energy difference is known as Voltage and is measured, naturally, in Volts. Because the potential energy of the Voltage is also measure of the force that would tend to move the charge, it is known also as the potential difference, or simply the potential, the “electromotive force” or the *e.m.f.* and these terms are used interchangeably, which can at times be confusing. Batteries are familiar voltage sources that rely on chemical means to store potential energy. For convenience, the units of Volts are defined in terms of other fundamental physical constants and units. One **Joule** of work is required to move one *Coulomb* of charge through a potential difference of 1 Volt. In practice, this means that a *Coulomb* is actually defined to set unit values of Volts and Joules. Voltage must always refer to the energy *difference* between two points. It is never actually correct to discuss the Voltage at a point, though you will often see such a statement. In those cases, the reference point is assumed implicitly, usually to refer to a “ground” or common point in an electrical circuit.

At the atomic level charges may become separated. In some molecules, such as salts like sodium chloride, the *electronegativity* of one atom (chloride) is so much greater than that of the other (sodium) that in a covalent atomic bond between these elements the electron or electrons are almost completely transferred from one atom to the other. Such bonds are dissociated easily in aqueous solution so that the individual atoms now become “*ions*” or charged particles. In water, the atoms of salts appear in ionic form, so that atoms of sodium, potassium, chloride, magnesium and many others move relatively freely of their oppositely charged complement. Not only atoms, but also molecules, can exist in ionic form, and many proteins, for example, carry a net negative charge. Of course some ions may be quite large and there may be physical impediments to their motion that result in different bulk properties for ions and small charges, such as electrons. These effects are significant in some circumstances, but in most of the

discussion that follows, and throughout most of this book, we can consider the properties of ions interchangeably with the properties of charge.

### *Current and Resistance*

The motion of charge is known as *current*; specifically, the current,  $i$ , is equal to the change in charge,  $Q$ , with time, so that:

$$i = dQ/dt.$$

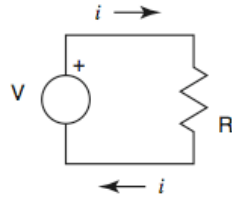
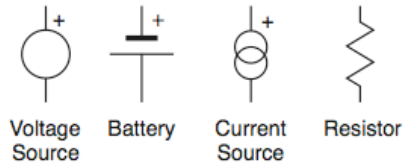
The international unit of current is the Ampere, or Amp. Currents flow whenever there is a Voltage difference across a medium with mobile charges, until the charge separation is eliminated (the Voltage is reduced to zero). While Voltage is a measure of *potential energy*, current represents *kinetic energy*. In practice, however, the motion of charge is never completely unimpeded; therefore, the current is limited. This restriction of charge movement is known as electrical **resistance** and is measured in units of Ohms. The greater the Voltage difference, not surprisingly, the greater the current that flows. Resistance is the simple proportionality between current and Voltage, which is expressed in the familiar form of “Ohm’s law”:

$$V = iR,$$

Where  $V$  is the Voltage,  $i$  is the current, and  $R$  the resistance. Materials whose resistance is extremely high are termed *insulators* and those whose resistance is low are called, *conductors*. Good insulators may have resistance of gigaOhms ( $10^9$  Ohms) or more, whereas good conductors, such as copper wire, will have resistance of microOhms. More accurately, we refer to **resistivity**, which is the measured resistance normalized by the area and length of a conductor, so that it is a material property. Most biological materials fall in a more intermediate range with resistances of thousands to millions of Ohms. In a perfect conductor, where the resistance is zero, the voltage at all points along the conductor is identical. In general, moving charge from a source of higher potential energy to lower (current flowing from positive to negative ends of a source) must result in energy *dissipation*. Resistors dissipate this energy as heat.

### *Kirchhoff’s Laws*

The path by which charge may flow between the positive and negative ends of a source of Voltage or charge is known as a *circuit*. Sources of current, Voltage, and resistance to current flow are presented in a set of standard symbols that are connected together in circuit diagrams. In such diagrams, lines represent perfect, zero resistance, conductors. Figure A1.01 shows these standard pictures. By convention, current is said to move from positive to negative potentials and would imply the motion of positive charges (in actuality, most currents in man made devices result from the motion of negative charge). Circuit diagrams frequently use arrows to indicate the direction of current flow. In the figure, note that the current *out* from the top, or positive end, of the Voltage source is identical to the current *in* to the bottom. The same is true of the resistor in the circuit and indeed in any element of a circuit: The current flow in is identical to the current flow out. This general behavior of nodes in a circuit is known as Kirchhoff’s current law (*KCL*).

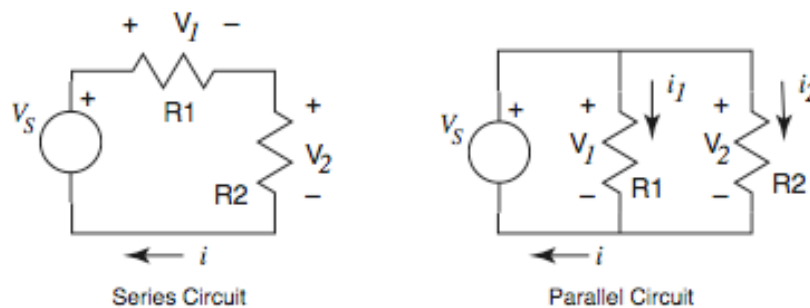


**Figure A.001.** Circuit symbols for common electrical devices and quantities.

Kirchhoff's current law has an analog (referred to as a “dual”) in describing Voltage differences in circuits. Kirchhoff's Voltage law (*KVL*) states that the sum of the Voltage differences around any closed loop in a circuit must be equal to zero. In applying *KVL* you must pay attention to the direction you travel in the loop, as discussed below.

### *Series and Parallel Connections*

When electrical elements are attached end-to-end they are connected “in series”, when the current inputs and outputs of multiple elements are held in common, the devices are connected in parallel. Figure A1.02 shows resistors connected to a Voltage source in series and in parallel. In the series circuit, *KCL* tells us that the current through all three devices is identical, while *KVL* informs us that the Voltages  $V_1$  and  $V_2$  have a sum equal to  $V_s$ . Applying *KCL* to the parallel circuit we see that at the two nodes,  $i_1 + i_2 = i$ . *KVL* tells us that traveling starting at the top and going clockwise around the loop that includes  $R_1$  and  $R_2$ ,  $V_2 - V_1 = 0$  and that  $V_2 = V_1$ . Likewise, in the loop made up of  $V_s$  and  $V_2$ ,  $V_2 - V_s = 0$ , implying that  $V_2 = V_s$ . The two Kirchhoff's laws, in combination, are extremely powerful organizing principles. Understanding them makes it possible to model the behavior of the overwhelming majority of electrical circuits and devices.



**Figure A.002.** Resistors in series and parallel circuits

Clearly, when the resistors are in series the same current flows through each. By Ohm's law, the voltage across  $R_1$  is equal to  $iR_1$ , and the Voltage that appears across  $R_2$  is  $iR_2$ . The Voltage that appears across the Voltage source is therefore the sum of the Voltages across each resistor, or  $i(R_1 + R_2)$  (Note that this is the same as the expression of *KVL* for this circuit:  $V_s = V_1 + V_2$ . The total resistance experienced by the Voltage source in

the series circuit is thus the sum of the individual resistors: resistance in series is the sum of the individual resistances.

In the parallel circuit, there are two paths for current to go through the resistors. Not surprisingly more current flows through the lower resistance path, and we can compute the individual currents by Ohm's law. Because the Voltage ( $V_1$  and  $V_2$ ) across these resistors is identical (they are connected by perfect conductors), the current,  $i_1$  in  $R_1$  is  $V/R_1$  and  $i_2$  is  $V/R_2$ . By KCL, the total current that flows from the Voltage source is equal to the sum of the currents through the two resistors:

$$i = i_1 + i_2. \quad (\text{Eq. 1})$$

Therefore the apparent resistance of the two resistors in parallel,  $R_{||}$ , is  $V/(i_1 + i_2)$ :

$$R_{||} = \frac{V}{i_1 + i_2} = \frac{V}{\left(\frac{V}{R_1} + \frac{V}{R_1}\right)} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_1}\right)}$$

$$R_{||} = (R_1 \cdot R_2)/(R_1 + R_2), \quad (\text{Eq. 2})$$

which is always less than either of the resistors alone.

### Energy Storage Devices

Though hundreds of different devices exist, we will consider just three other circuit elements: *batteries*, *capacitors* and *inductors*.

A battery is a familiar device that is an approximation to an ideal Voltage source. Physically, batteries produce voltage difference through the chemical separation of charges. As current is allowed to flow from one pole of the battery to the other, charges move back together releasing energy, generally into the circuit that connects the poles of the battery. In the circuit of A1.003 (a), a wire connects the positive and negative pole of a battery. If the wire were "ideal", having zero resistance, we would expect that all of the charge would flow instantaneously from the positive to the negative pole of the battery, such that the current,  $dQ/dt$ , would be infinite. Of course this doesn't actually happen. Instead, it turns out that all real batteries show some resistance to current flow, which limits the current. The circuit of A1.003 (b) shows a practical and realistic model of the battery as an ideal Voltage source in series with a resistor. (This combination is known as the **Thévenin equivalent**). Many of the electrical properties of neurons and other excitable cells may be accurately modeled in this manner.

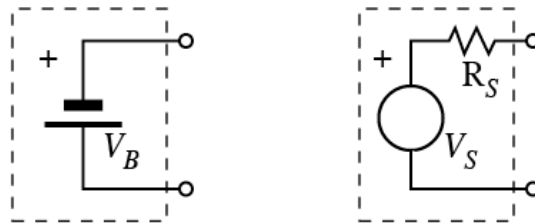
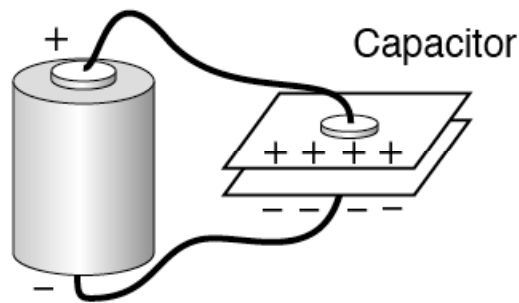


Figure A.003. Thévenin equivalent of a battery.

### Capacitors

When two conductors are separated from one another by an insulator, and a source of current is applied (Figure A1.004), positive charge will accumulate on one side of the insulator and negative charge on the other, resulting in a buildup of Voltage. As long as the charge keeps flowing the Voltage will increase steadily, and without limit.

Analogously, if a Voltage source is applied across the separated conductors, charge will build (immediately, in the case of ideal conductors) of such a magnitude that the potential energy difference is equal to the Voltage of the source. When the sources are removed, however, the charge difference, and therefore the Voltage difference, is retained across the insulating boundary, thus storing potential energy. A device made of conductors – typically in the form of thin films – separated by an insulating layer is known as a *capacitor*. However, capacitance will exist between any conductors separated by an insulating layer or material. Unlike resistors, capacitors store energy, rather than dissipate it.



**Figure A.004.** Physical structure of a capacitor.

The capacitance,  $C$ , is measured as the ratio of the charge across the capacitor to the applied Voltage:

$$Q = CV. \quad (\text{Eq. 3})$$

The circuit symbol for a capacitor is shown in figure A1.005, and represents two separated conductive plates.



**Figure A.005.** Circuit symbol for a capacitor.

The unit of capacitance is the **Farad**. A large capacitance, achieved by having a very small insulating gap, means that a relatively small charge results in a large Voltage potential difference, chiefly because the attractive forces between positive and negative charge are very large over short distances. Taking the first derivative of equation 1, we see that the current is proportional to the Voltage change over time.

$$i_c = C dV/dt. \quad (\text{Eq. 4})$$

In a sense, this appears similar to Ohm's law, except that now the current is proportional to the rate of change of the Voltage, rather than the Voltage alone, as it is in a resistor. The capacitor, in this case, takes the place of the resistor, but one whose resistance depends on the rate of change of the Voltage. Rather than resistance, the term

*impedance*, measured still in Ohms, is used to describe this behavior. In effect, this means that while an insulating layer does not pass constant current, time-varying currents may be passed. The units of Farads are defined such that a Voltage that changes by 1 V/s, when applied across a 1 Farad capacitor, will result in the flow of 1 ampere (a very large amount) of current. Most often, capacitors used in electronic circuits have capacitance of a few microfarads ( $\mu\text{F}$ ).

Specifically, consider applying a sinusoidally varying voltage across the capacitor:

$$V(t) = A\cos(\omega t). \quad (\text{Eq. 5})$$

The current through the capacitor, by equation 2, would therefore be:

$$i_C = -\omega C A \sin(\omega t), \quad (\text{Eq. 6})$$

that is, a sinusoidal current with a  $90^\circ (= \pi/4)$  phase *lead* with respect to the Voltage. The term,  $\omega$ , is the frequency, and this frequency-dependent resistance, measured in Ohms, is given the term *impedance*. Specifically, the impedance goes down as the frequency goes up. Capacitors pass greater current at the same Voltage when the frequency is higher. They are like resistors whose resistance decreases with frequency. Unlike resistors however, which dissipate energy as heat, the capacitor only stores energy, by converting between potential energy in the form of Voltage, and kinetic energy in the form of current.

More generally, consider Voltages of the form:

$$V(t) = A e^{st}. \quad (\text{Eq. 7})$$

In this case, the current through the capacitor will be:

$$i_C = s C A e^{st}. \quad (\text{Eq. 8})$$

The latter is a particularly useful form. It shows that the proportionality between Voltage and current in a capacitor is given as:

$$V_c / i_c = 1/sC. \quad (\text{Eq. 9})$$

Ohms law tells us that the magnitude of this proportionality is the resistance, which we now see is proportional to  $s$ . The exponential term,  $e^{st}$ , simply drops out of the equation. No assumptions were made about  $A$  or  $s$ , however, except that they are constant with respect to  $t$ . This solution form to the differential equation 4 is known as the **Laplace transform**, and is an important solution method in all forms of linear systems.

### *Complex Representation of Signals*

Most generally, we can use complex or imaginary values for  $s$  (and/or  $A$  for that matter) in equation 9, where  $s$  is known as the “complex frequency.” In electrical engineering it is common practice to use the convention that  $j = \sqrt{-1}$  in order to avoid confusion with the use of  $i$  to represent current; we will use that convention for the remainder of this

section. When  $s$  is purely imaginary, with the value  $s = j\omega$  and  $A$  is real, we can take advantage of Euler's relation, that:

$$Ae^{st} = Ae^{j\omega t} = A[\cos(\omega t) + j\sin(\omega t)]. \quad (\text{Eq. 10})$$

We can apply a sinusoidally varying voltage at the frequency  $\omega$ ,  $V = \cos(\omega t)$ , across the capacitor. Note that the  $V$  is the real part of  $Ae^{j\omega t} = \mathcal{R}\{Ae^{st}\}$ .

Substituting into equation 8:

$$\begin{aligned} i_c &= sCAe^{st} = j\omega C A[\cos(\omega t) + j\sin(\omega t)] \\ &= j\omega C A \cos(\omega t) - \omega C A \sin(\omega t). \end{aligned} \quad (\text{Eq. 11})$$

We are interested only the *real* part of the current, of course, so equation 11 simplifies to  $i_c = -\omega C A \sin(\omega t)$ , the result of equation 6 above. Looking again at equation 9, when  $s = j\omega$  the ratio of the voltage to the current is  $1/j\omega C$ .

### *Generalization of Ohm's law*

The ratio of a complex voltage,  $\mathbf{V}$ , to a complex current,  $\mathbf{I}$ , is represented by the *impedance*,  $\mathbf{Z}$ . Because sinusoidally varying signals are conveniently represented by complex numbers (and, as we will see when we discuss the Fourier transform in a separate section, this can be a generalized representation of most any signal we encounter), this is an exceptionally powerful formalization. For complex signals, Ohm's law states simply that:

$$\mathbf{V}/\mathbf{I} = \mathbf{Z}. \quad (\text{Eq. 12})$$

The magnitude of the impedance is given in units of Ohms, like resistance, but in general also incorporates a phase shift. For capacitors, as noted above, the impedance is  $1/j\omega C$ . Applying Voltage as a cosine wave results in a current that is an inverted sine wave – a  $90^\circ$  phase shift. In general, multiplication by  $j$  represents a  $90^\circ$  phase shift.

As we noted above, the voltage and current relationships for most electrical circuits (specifically, *linear* electrical circuits), can be found through the application of Ohm's law coupled with KVL and KCL. The Laplace transform solution, as in equation 9, allows this to be generalized readily to the analysis of devices like capacitors, which store energy.

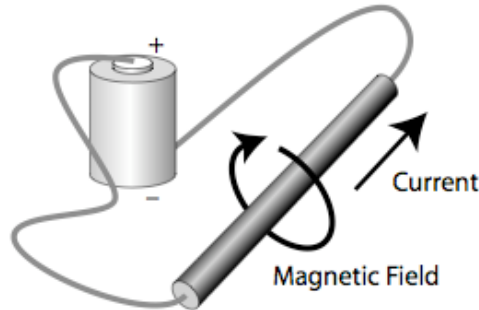
One should be a little bit careful here concerning the values of  $\omega$ , which are in units of radians/second or angular frequency. In common use we generally speak of frequencies in cycles/second or "Hertz". As one cycle is  $2\pi$  radians,  $\omega = 2\pi f$ , where the letter  $f$  denotes the frequency in cycles/second. When the input voltage is a sinusoid with frequency,  $f$ , equation 9 may be written as:

$$V_c / i_c = 1/j2\pi f C. \quad (\text{Eq. 13})$$

### *Magnetism*

While we will give neither a historical review, nor a physical derivation, it is well established that:

- 1) the flow of electrical current results in the creation of a magnetic field, whose strength is proportional to the magnitude of the current, and whose direction is circumferential to the conductor (Fig A1.004)



**Figure A.006.** Magnetic field induced by the flow of current through a conductor.

- 2) a time-varying magnetic field results charged in particles experiencing an electromotive force, or electrical Voltage field. The magnitude of the *e.m.f.* is proportional to the rate of change of the magnetic field.

$$e.m.f. = V = -dB/dt^1, \quad (\text{Eq. 14})$$

The minus sign in equation 14 reminds us that the magnetic field opposes the current flow.

The thoughtful reader might notice that statements 1 and 2 are equivalent in a relativistic sense. Charge moving through a field (current) is equivalent to a field moving with respect to the charge. In practice, physicists treat electricity and magnetism essentially as different aspects of the same phenomenon.

Imagine an isolated electron in space. With respect to an observer passing by, it appears as a current flowing in a direction opposite to the observer's direction of travel. Within the observer's frame of reference, there will be a magnetic field created. If the moving observer had a permanent magnet in hand, it would experience a magnetic force to align with the field created by the isolated electron. On the other hand, from the perspective of the electron, the observer's magnet would be experienced as a passing (changing) magnetic field, resulting in an electromotive force. These reciprocities are not just curiosities, but important concepts to hang on to when we discuss radio signals later.

## *Inductors*

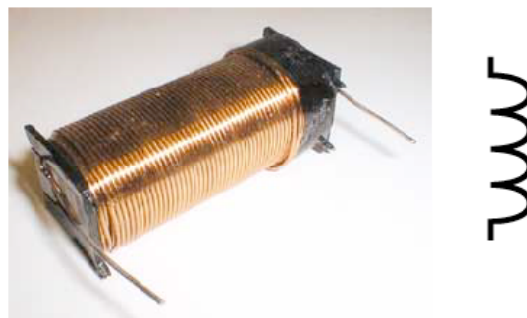
---

<sup>1</sup> Strictly speaking, this equation is not accurate, as the electromotive forces appear as a vector field. The complete form of equation 14 is:  $\nabla \times E = -\partial B / \partial t$ . For our purposes, there is no need to consider the math in this detail.

The final electrical device we will consider is the *inductor*. Physically, an inductor consists of nothing more than a coil of wire (in the very high *radio* frequencies, even ideal conductors have inductive properties – a fact of some importance when trying to design MRI devices).

Suppose now that we run a sinusoidally varying current,  $i = A\sin(\omega t)$ , through a wire. By statement 1, above, this results in the creation of a time-varying magnetic field. By statement two, we would expect that this would in turn cause charges in the wire to experience an electromotive force. The *direction* of this force, however, is opposite to the direction of motion of the charge (otherwise, of course, a positive feedback would be in place and the process would result in the creation of energy.) In effect, this means that the magnetic field created in a wire by time-varying currents opposes this current flow. Further, because the magnetic field is proportional to the time derivative of the current, it impedes the current flow more strongly as the frequency increases.

Forming our conductors into coils results in the more efficient coupling of the magnetic fields to the wire itself, as the magnetic fields induced in each loop of wire pass through the adjacent loops; the circuit symbol for an inductor is a cartoon for such a series of loops (Figure A.007).



**Fig. A.007.** (*Left*) Real-world inductor formed as a spool of insulated copper wire. (*Right*) Circuit symbol for an inductor. (FROM: <http://www.goldmine-elec-products.com/images/G13205B.jpg>)

The proportionality between the rate of change of current and Voltage is called the *inductance* and is measure in units of Henrys, conventionally denoted with the letter, L (for Lorentz). This is summarized in equation 15.

$$V_L = L di_L/dt. \quad (\text{Eq. 15})$$

Note the close parallel between this and equation 4; they differ only by the substitution of Voltage by current (and current by Voltage), and by the use of the letter L to denote inductance, instead of C to indicate current. We can use the same means, the Laplace transform, to solve both equations.

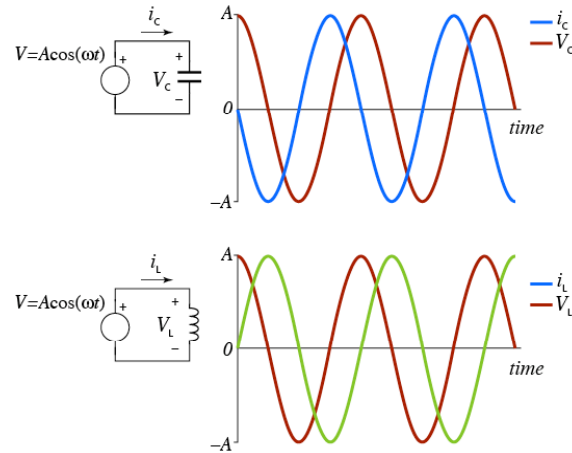
Let  $i(t) = Ae^{st}$ . Therefore  $di/dt = Ase^{st}$ . It follows that:

$$V_L = L di_L/dt = sL Ase^{st} \text{ and}$$

$$V_L/i_L = sL$$

$$\mathbf{V}_L/\mathbf{I}_L = \mathbf{Z}_L = j\omega L. \quad (\text{Eq. 16})$$




Comparing equations 16 and 9, we see an important difference. Namely that  $s$  appears in the numerator rather than the denominator. This implies that in an inductor the impedance *increases* with frequency. An inductor acts like a resistor whose resistance increases with frequency. Further, in an inductor, the current *lags* (rather than leads) the Voltage by  $90^\circ$ . Figure A.007 shows the phase relationships between Voltage and current in inductors and capacitors.



**Fig A1.008.** Voltage and current phase relationships in capacitors (top) and inductors (bottom) when sinusoidal voltages are applied.

Perhaps the most salient feature of this figure is that the current in the capacitor and the inductor are  $180^\circ$  out of phase with each other. While these phase relationships are always present, you should be aware that the amplitude relationships depend on the frequency and on the values of the inductance of capacitance.

Summarizing then, the impedance is a complex quantity, whose *magnitude* is measured in Ohms. The real part of the impedance is the *resistance*, whereas the imaginary part is known as the *reactance*.

Resistor		$\mathbf{Z}_R = R + 0j$
Capacitor		$\mathbf{Z}_C = 0 + 1/j\omega C$
Inductor		$\mathbf{Z}_L = 0 + j\omega L$

As you can see, the impedance of resistors is purely real, while that of capacitors and inductors is purely imaginary. The real part of the resistance is dissipative of energy, converting the kinetic energy (current) or potential energy (Voltage) to heat. The imaginary part of the impedance does not dissipate power, storing energy instead, either as potential energy in the charge across the capacitor or as kinetic energy in the maintained current flow in inductors (which impede changes in current).

### *Impedance in Circuits*

Fortunately, we will have little need to perform complicated circuit analysis in this text. Nevertheless, it is important to recognize the power of complex analysis to understanding sophisticated circuits. The key realization is that we can treat the impedance just like resistance in predicting the current and Voltage relationships in

electrical circuits. Specifically, not only to KVL and KCL apply to all nodes and loops in a circuit carrying complex signals, but impedances sum like resistance:

When arranged in series:  $\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_1 + \dots + \mathbf{Z}_N$ .

When impedances are placed in parallel:  $\mathbf{Z} = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \dots + \frac{1}{\mathbf{Z}_N}}$ .

If an inductor and capacitor are placed in series, there will be a frequency at which the reactance of the two devices will be equal. At this point, because the phases are opposite one another, no current will pass. This condition is known as *resonance*. Similarly, when the devices are placed in parallel, if Voltage is applied at this same frequency the total reactance will drop to zero and *infinite* current will pass.

### *Electrical Components, Neurons and MRI*

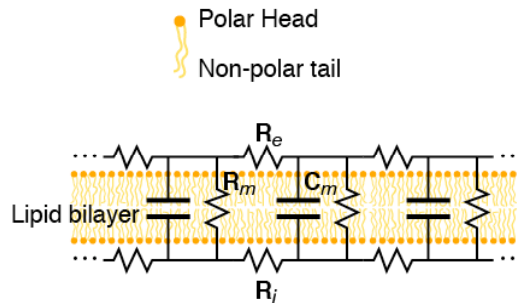
It is easy to see how capacitance and resistance might apply to the electrical properties of the neuron. We will consider these in some detail, because the interaction of tissues MRI necessarily includes current flows. Further, with increasing interest in both the direct measurement of electrical signals by MRI, or in combining both electrical and imaging acquisitions (such as EEG with MRI), it is important to understand the electrical characteristics of the tissues.

Whereas current in conductors such as copper is carried more or less directly in electrons, electrical current in biological systems is carried principally on charged molecules (ions) that must physically move through space for current to flow. Obvious physical factors, such as steric hindrance, impede the motion of these ions and result in electrical resistance. The small ion channel pores that cross the cell membrane of neurons allow more or fewer charged ions to pass through, and thus act as resistors of variable magnitude.

The **lipid** bilayer, of which the neuronal cell membrane is made, is a very effective insulator that prevents the movement of charged particles, because such particles are effectively excluded from the non-polar hydrocarbon tails that make up its interior. The separation of charge therefore confers *capacitance* to the cell membrane. This capacitance is actually rather large. From its thickness of about 7 to 10 nm it is possible to calculate the capacitance as approximately 1  $\mu\text{F}/\text{cm}^2$  of membrane. While the resistivity of membrane lipids is quite high, the membrane is somewhat deformable, and with the resting potential across a neuron of about 0.1V the compressive stress created by the separated charges is very high, on the order of  $10^{12}$  Newtons/ $\text{cm}^2$ , or  $14.5 \times 10^9$  lbs/ $\text{in}^2$ . Thus, the capacitance of the cell membrane could be expected to increase as the transmembrane Voltage is increased, as the membrane is made thinner.

In the face of such overwhelming forces, some charge passes through the membrane, of course, either through the lipid bilayer itself, or through defects in the membrane created by transmembrane proteins or channels. This leads to the standard electrical model of a lipid bilayer of figure A1.009. In that figure,  $R_e$  refers to the resistance of the extracellular fluid,  $R_i$  is the intracellular resistance,  $R_m$  is the transmembrane resistance and  $C_m$  is the transmembrane capacitance. The electrical symbols are shown as lumped

elements, but in reality, we should think of these as existing in the limit of infinitely small length. The physical structure of the bilayer is shown in a cartoon beneath.

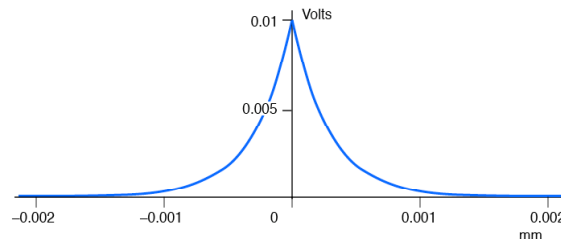


**Figure A1.009.** Equivalent circuit for passive membrane properties of a neuron. The bilayer that makes up the cell membrane consists of a series of lipid molecules that have a polar (charged) head that is hydrophilic and non-polar tails that are hydrophobic. The non-polar region that makes up the interior of the bilayer is essentially impervious to ions. Though shown as lumped elements, the resistors and capacitors that make up this model are actually continuously distributed along the membrane.

If the Voltage across the membrane is increased in a single location by an amount  $V_0$ , the ladder of resistive elements will cause the transmembrane Voltage,  $V_m$ , to decrease exponentially with distance from the local increase in potential; the transmembrane Voltage can be calculated as follows:

$$V_m = V_0 e^{-x/\lambda}, \quad (\text{Eq. 17})$$

where  $\lambda$  is the so-called “length constant” determined by  $R_m$ ,  $R_e$ , and  $R_i$ , and  $x$  is the distance along the neuron. Figure A1.010 shows how the transmembrane potential varies with distance when a potential of 0.1 Volts is applied locally across a neuron having a length constant of 200  $\mu\text{m}$ . In practice  $\lambda$  varies greatly for different neurons and parts of neurons, because of differences both in the intracellular constituents that alter  $R_i$ , and the presence of myelination, which greatly increases  $R_m$ . Measured values of  $\lambda$  range from a few microns to a millimeter or more.



**Figure A1.010.** Spatial spread of transmembrane potential along the neuron for a 0.01 Volt potential placed at an arbitrary location 0, corresponding to a neuron with a length constant,  $\lambda$ , of 200  $\mu\text{m}$ .

The capacitance that appears in Figure A1.009 also results in a time dependence for transmembrane Voltages. If the charge across one of the lumped capacitor elements in that figure were instantaneously increased, it would ultimately leak out across the resistive paths, resulting in an exponential decrease in the potential difference over time. The time constant,  $\tau$ , for this decay is approximately the product of  $R_m$  and  $C_m$ . For vertebrate neurons,  $\tau$  ranges from about 0.5 milliseconds to 5 milliseconds.

Inductive characteristics are much less obvious in neurons, though there are a few cases where they become important. Specifically, the transient flow of currents during the action potential results in the creation of a small magnetic field (very small, in fact, measurable in units of  $10^{-15}$  Tesla (femtoTesla)). The important brain imaging technique of magnetoencephalography is based on the detection of these tiny signals magnetic signals. Several investigators have attempted to detect neuronal activity using MRI by making scans sensitive to the tiny spatial variations in magnetic field that such currents create. As of this writing, the method is fraught with controversy and methodological challenges, however.

Inductance, however, is a very important factor in the design of devices used in MRI. For example, the magnets used for MRI are made by moving large currents through coils of wire. Each turn of the wire results in the creation of a magnetic field and the more turns, and the more current, the greater will be the magnetic field strength. In human imaging magnets several kilometers of wire are wound into coils to create field strengths of several Tesla. Currents of several hundred Amperes are typical. Any resistance at all would result in the dissipation of the kinetic energy from the charge motion into heat. MRI devices, however, usually are made by cooling special materials, such as niobium and tin, to a few degrees above absolute zero, where they become *superconductive* and have no resistance to current flow.

Recently, it has been demonstrated that biological currents through the brain appear to be affected by significant inductance of unknown origin (that is, at higher frequencies the current flow is reduced) [Akhtari, et al.]. These unexpected effects are likely to become very significant in the interpretation of electrical signals at the surface of the brain or head (electroencephalography or EEG) as they may affect our ability to localize the signal sources in three dimensions.

While this appendix is not intended as a complete course in electricity and magnetism, it is designed to offer the minimum knowledge needed to understand the material in the main text. Electrical circuit models of resistors, inductors and capacitors are an excellent basis from which to analyze linear systems of enormous complexity. Familiarity with Ohms law, KVL, KCL and the impedance properties of capacitors and inductors is sufficient to model many dynamical biological systems and is essential to a deep understanding of magnetic resonance imaging devices. The reader is encouraged to find more information in the excellent texts below:

## SUGGESTED READING

General Principles of Electricity and Electronics

Horowitz and Hill, "The Art of Electronics" (1)

Electrical Properties of Neurons

W. Rall, in *Handbook of Physiology* E. Kandel, Ed. (American Physiological Society, Bethesda, 1977), vol. 1, pp. 39-37. (2)

W. Rall, in *Methods in Neuronal Modeling* C. Koch, I. Segev, Eds. (MIT Press, Cambridge, 1989) pp. 8-62. (3)

1. P. Horowitz, W. Hill, *The art of electronics* (Cambridge University Press, Cambridge [England]; New York, ed. 2nd, 2001), pp. xxiii, 1125 p.
2. W. Rall, in *Handbook of Physiology* E. Kandel, Ed. (American Physiological Society, Bethesda, 1977), vol. 1, pp. 39-37.
3. W. Rall, in *Methods in Neuronal Modeling* C. Koch, I. Segev, Eds. (MIT Press, Cambridge, 1989) pp. 8-62.